MULTIMEDIA UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

# FINAL EXAMINATION

**TRIMESTER 2, 2017/2018**

## BMS 1024 – MANAGERIAL STATISTICS

(All sections / Groups)

3 March 2018

9.00 a.m – 11.00 a.m

(2 Hours)

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**INSTRUCTIONS TO STUDENTS**

1. This question paper consists of **TEN (10)** printed pages with:
   **Section A**: Ten (10) multiple choice questions (20%)
   **Section B**: Three (3) structured questions (80%)

2. Answer **all** questions.

3. Answer **Section A** in the multiple-choice answer sheet provided and **Section B** in the answer booklet provided.

4. Formula and Statistical tables are attached at the end of the question paper.

5. Students are allowed to use non-programmable scientific calculators with no restrictions.

## SECTION A: MULTIPLE CHOICE QUESTIONS (20 MARKS)

**There are TEN (10) questions in this section. Answer ALL questions on the multiple choice answer sheet.**

1. In measures of central tendency, sample statistic is denoted by
   A. $\mu$
   B. $\beta$
   C. $\bar{x}$
   D. $\alpha$

2. In a negative skewed distribution, the order of mean and median is
   A. mean = median
   B. mean < median
   C. mean > median
   D. mean ≠ median

3. The collection of one or more outcomes from an experiment is called
   A. probability
   B. event
   C. discrete random variable
   D. z- value

4. The joint probability is
   A. the likelihood of two events happening to get the probability
   B. the likelihood of an event happening given that another event has already happened
   C. based on two mutually exclusive events
   D. also called Prior probability

5. Which of the following is not a condition of the binomial distribution?
   A. Only 2 possible outcomes
   B. have constant probability of success
   C. must have at least 3 trials
   D. trials must be independent

6. If value of x for normal distribution is 35, the mean of normal distribution is 65 and the standard deviation is 25 then standardized random variable, z is
   A. -1.5
   B. -1.4
   C. -1.7
   D. -1.2

**Continued...**

7. If you reject a true null hypothesis, what does this mean?
   A. You have made a correct decision.
   B. You have made a Type I error.
   C. You have made a Type II error.
   D. You have increased the power of a test.

| Commodity | 2000 | | 2017 | |
|---|---|---|---|---|
| | Price (RM) | Quantity | Price (RM) | Quantity |
| A | 2 | 20 | 3 | 21 |
| B | 18 | 3 | 36 | 2 |
| C | 3 | 18 | 4 | 23 |

Table 1: Demand for Commodity A, B and C for year 2000 and 2017

8. Based on Table 1, what is the unweighted aggregate price index for 2017 with 2000 as the base year?
   A. 186.96
   B. 180.96
   C. 176.96
   D. 170.96

9. Based on Table 1, what is the Laspeyres aggregate price index for 2017 with 2016 as the base year?
   A. 152.16
   B. 162.16
   C. 158.20
   D. 158.20

10. Based on Table 1, what is the Paasche aggregate price index for 2017 with 2016 as the base year?
    A. 152.38
    B. 153.38
    C. 150.38
    D. 154.38

**Continued...**

## SECTION B: STRUCTURED QUESTIONS (80) MARKS)

**There are THREE (3) questions in this section. Candidates MUST answer ALL THREE questions.**

### Question 1 (25 Marks)

a) From a set of 3 females and 4 males' students, three students were selected for inter university entrepreneurship fest. Let X be the random variable that denotes the number of girls selected.

i) Construct the probability distribution of X. (5 marks)

ii) Find the variance of $X$. (5 marks)

b) One of the audit firm in Malaysia audits 5% of all companies every year. The companies selected for auditing in any one year are independent of the previous year's selection. What is the probability that one of the company listed will be selected for auditing exactly twice in the next 5 years? (5 marks)

c) Suppose a famous seafood restaurant can expect two customers every 10 minutes, on average. Calculate the probability that less than four customers will enter the restaurant in a 30-minute period? (5 marks)

d) An investment return is normally distributed with a mean of 6% and a standard deviation of 3%. Determine the probability of losing money and sketch the area. (5 marks)

### Question 2 (25 Marks)

a) Suppose that in a large city the annual income of real estate agents is normally distributed with a standard deviation of \$40. A random sample of 15 real estate agents was asked to report their annual income (in \$1000) and listed as below:

| | | | | |
|---|---|---|---|---|
| 180 | 130 | 150 | 165 | 90 |
| 130 | 120 | 60 | 200 | 180 |
| 80 | 240 | 210 | 150 | 125 |

i) Construct a 95% confidence interval estimate of the population mean. Express your answer in 2 decimal places. (6 marks)

ii) Interpret your answer in (i) (2 marks)

iii) At 90% confidence level, how large a sample should be selected if they want the estimate to be within \$10 of the population mean? (6 marks)

**Continued...**

b) The weight of children age 3 years old are normally distributed with a mean of 15 kg. A researcher claims that the weight of children who were sent to nursery will have a lower weight compared to the population mean. To test the researcher's claim, a random sample of 40 children were examined. The mean weight in this sample was 12.5 kg with a standard deviation of 2 kg. Would you conclude that the researcher's claim is true at a 1% significance level? (11marks)

## Question 3 (30 Marks)

a) In a meat packing factory, a new machine will pack faster on the average than the machine currently used. To test that hypothesis, the times it takes each machine to pack ten cartons are recorded. The results, in seconds, are shown in the following table. Use significance level: $\alpha = 0.05$. (25 marks)

| | New Machine | Old Machine |
|---|---|---|
| 1 | 32.1 | 32.7 |
| 2 | 31 | 33.6 |
| 3 | 31.3 | 33.8 |
| 4 | 31.8 | 33.3 |
| 5 | 32.4 | 32.5 |
| 6 | 32.8 | 33.5 |
| 7 | 33.2 | 33.1 |
| 8 | 32.3 | 31.7 |
| 9 | 31.8 | 34 |
| 10 | 32.7 | 34.1 |

b) The sales of a company (in million dollars) for each year are shown in the table below:

| Year | 2013 | 2014 | 2015 | 2016 | 2017 |
|---|---|---|---|---|---|
| Sales | 12 | 19 | 29 | 37 | 45 |

(Let y = sales and x = number of years)

Given the least square regression line for the above data is $y = 8.4x + 11.6$, estimate the sales of the company in 2020.
(Use 2013 as the starting year, $x = 0$). (5 marks)

**End of Page**

## STATISTICAL FORMULAE

### A. DESCRIPTIVE STATISTICS

Mean $(\bar{x}) = \dfrac{\sum_{i=1}^{n} X_i}{n}$

Standard Deviation $(s) = \sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{(\sum_{i=1}^{n} X_i)^2}{n(n-1)}}$

Coefficient of Variation $(CV) = \dfrac{\sigma}{\bar{X}} \times 100$

Pearson's Coefficient of Skewness $(S_k) = \dfrac{3(\bar{X} - Median)}{s}$

### B. PROBABILITY

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A)× P (B) if A and B are independent

P (A | B) = P (A and B) ÷ P (B)

**Poisson Probability Distribution**

If $X$ follows a Poisson Distribution, $P(\lambda)$ where $P(X = x) = \dfrac{e^{-\lambda}\lambda^x}{x!}$

then the mean = $E(X) = \lambda$ and variance = $VAR(X) = \lambda$

**Binomial Probability Distribution**

If $X$ follows a Binomial Distribution $B(n, p)$ where $P(X = x) = {}^nC_x p^x q^{n-x}$

then the mean = $E(X) = np$ and variance = $VAR(X) = npq$ where $q = 1 - p$

**Normal Distribution**

If $X$ follows a Normal distribution, $N(\mu, \sigma)$ where $E(X) = \mu$ and $VAR(X) = \sigma^2$

then $Z = \dfrac{X - \mu}{\sigma}$

### C. EXPECTATION AND VARIANCE OPERATORS

$E(X) = \sum[X \bullet P(X)]$

$VAR(X) = E(X^2) - [E(X)]^2$ where $E(X^2) = \sum[X^2 \bullet P(X)]$

If $E(X) = \mu$ then $E(cX) = c\mu$, $E(X_1 + X_2) = E(X_1) + E(X_2)$

If $VAR(X) = \sigma^2$ then $VAR(cX) = c^2\sigma^2$,

$VAR(X_1 + X_2) = VAR(X_1) + VAR(X_2) + 2\,COV(X_1, X_2)$

where $COV(X_1, X_2) = E(X_1 X_2) - [E(X_1)\,E(X_2)]$

## D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

**(100 – α) % Confidence Interval for Population Mean (σ Known) =** $\mu = \overline{X} \pm Z_{\alpha/2}\left(\sigma / \sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Mean (σ Unknown) = $\mu = \overline{X} \pm t_{\alpha/2, n-1}\left(s / \sqrt{n}\right)$

(100 – α)% Confidence Interval for Population Proportion $= \hat{p} \pm Z_{\alpha/2}\sigma_{p^\wedge}$

Where $\sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size Determination for Population Mean $= n \geq \left[\frac{(Z_{\alpha/2})\sigma}{E}\right]^2$

Sample Size Determination for Population Proportion $= n \geq \frac{(Z_{\alpha/2})^2\, \hat{p}(1-\hat{p})}{E^2}$

Where E = Limit of Error in Estimation

## E. HYPOTHESIS TESTING

**One Sample Mean Test**

| Standard Deviation (σ) Known | Standard Deviation (σ) Not Known |
|---|---|
| $Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ | $t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$ |

**One Sample Proportion Test**

$z = \frac{\hat{p} - p}{\sigma_p}$ where $\sigma_p = \sqrt{\frac{p(1-p)}{n}}$

Two Sample Mean Test

| Standard Deviation (σ) Known | Standard Deviation (σ) Not Known |
|---|---|
| $z = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2 / n_1 + \sigma_2^2 / n_2}}$ | $t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_p^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$ |

***Two Sample Proportion Test***

$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{p(1-p)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$ where $p = \frac{X_1 + X_2}{n_1 + n_2}$

*where $X_1$ and $X_2$ are the number of successes from each population*

## F. REGRESSION ANALYSIS

**Simple Linear Regression**

**Population Model:** $Y = \beta_0 + \beta_1 X_1 + \varepsilon$

**Sample Model:** $\hat{y} = b_0 + b_1 x_1 + e$

**Correlation Coefficient**

$$r = \frac{\sum XY - \left[\frac{\sum X \sum Y}{n}\right]}{\sqrt{\left[\sum X^2 - \left(\frac{(\sum X)^2}{n}\right)\right]\left[\sum Y^2 - \left(\frac{(\sum Y)^2}{n}\right)\right]}} = \frac{COV(X,Y)}{\sigma_X \sigma_Y}$$

**ANOVA Table for Regression**

| Source | Degrees of Freedom | Sum of Squares | Mean Squares |
|---|---|---|---|
| Regression | *1* | SSR | MSR = SSR/1 |
| Error/Residual | $n-2$ | SSE | MSE = SSE/($n-2$) |
| Total | $n-1$ | SST | |

**Test Statistic for Significance of the Predictor Variable**

$$t_i = \frac{b_i}{S_{b_i}} \quad \text{and the critical value} = \pm t_{\alpha/2,(n-p-1)}$$

*Where p= number of predictor*

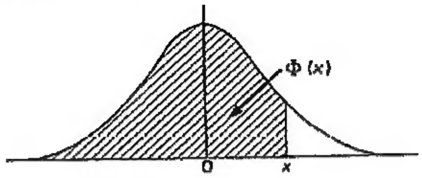
## G. INDEX NUMBERS

| | |
|---|---|
| **Simple Price Index** $P = \frac{p_t}{p_0} \times 100$ | **Laspeyres Quantity Index** $P = \frac{\sum p_0 q_t}{\sum p_0 q_0} \times 100$ |
| **Aggregate Price Index** $P = \frac{\sum p_t}{\sum p_0}(100)$ | **Paasche Quantity Index** $P = \frac{\sum p_t q_t}{\sum p_t q_0} \times 100$ |
| **Laspeyres Price Index** $P = \frac{\sum p_t q_0}{\sum p_0 q_0} \times 100$ | **Fisher's Ideal Price Index** $\sqrt{(Laspeyres\ \ Price\ Index)(Paa\ sche\ Price\ Index)}$ |
| **Paasche Price Index** $P = \frac{\sum p_t q_t}{\sum p_0 q_t} \times 100$ | **Value Index** $V = \frac{\sum p_t q_t}{\sum p_0 q_0} \times 100$ |

## B. STATISTICAL TABLE

### TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to $x$. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.




| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0·00 | 0·5000 | 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·97725 |
| ·01 | ·5040 | ·41 | ·6591 | ·81 | ·7910 | ·21 | ·8869 | ·61 | ·9463 | ·01 | ·97778 |
| ·02 | ·5080 | ·42 | ·6628 | ·82 | ·7939 | ·22 | ·8888 | ·62 | ·9474 | ·02 | ·97831 |
| ·03 | ·5120 | ·43 | ·6664 | ·83 | ·7967 | ·23 | ·8907 | ·63 | ·9484 | ·03 | ·97882 |
| ·04 | ·5160 | ·44 | ·6700 | ·84 | ·7995 | ·24 | ·8925 | ·64 | ·9495 | ·04 | ·97932 |
| 0·05 | 0·5199 | 0·45 | 0·6736 | 0·85 | 0·8023 | 1·25 | 0·8944 | 1·65 | 0·9505 | 2·05 | 0·97982 |
| ·06 | ·5239 | ·46 | ·6772 | ·86 | ·8051 | ·26 | ·8962 | ·66 | ·9515 | ·06 | ·98030 |
| ·07 | ·5279 | ·47 | ·6808 | ·87 | ·8078 | ·27 | ·8980 | ·67 | ·9525 | ·07 | ·98077 |
| ·08 | ·5319 | ·48 | ·6844 | ·88 | ·8106 | ·28 | ·8997 | ·68 | ·9535 | ·08 | ·98124 |
| ·09 | ·5359 | ·49 | ·6879 | ·89 | ·8133 | ·29 | ·9015 | ·69 | ·9545 | ·09 | ·98169 |
| 0·10 | 0·5398 | 0·50 | 0·6915 | 0·90 | 0·8159 | 1·30 | 0·9032 | 1·70 | 0·9554 | 2·10 | 0·98214 |
| ·11 | ·5438 | ·51 | ·6950 | ·91 | ·8186 | ·31 | ·9049 | ·71 | ·9564 | ·11 | ·98257 |
| ·12 | ·5478 | ·52 | ·6985 | ·92 | ·8212 | ·32 | ·9066 | ·72 | ·9573 | ·12 | ·98300 |
| ·13 | ·5517 | ·53 | ·7019 | ·93 | ·8238 | ·33 | ·9082 | ·73 | ·9582 | ·13 | ·98341 |
| ·14 | ·5557 | ·54 | ·7054 | ·94 | ·8264 | ·34 | ·9099 | ·74 | ·9591 | ·14 | ·98382 |
| 0·15 | 0·5596 | 0·55 | 0·7088 | 0·95 | 0·8289 | 1·35 | 0·9115 | 1·75 | 0·9599 | 2·15 | 0·98422 |
| ·16 | ·5636 | ·56 | ·7123 | ·96 | ·8315 | ·36 | ·9131 | ·76 | ·9608 | ·16 | ·98461 |
| ·17 | ·5675 | ·57 | ·7157 | ·97 | ·8340 | ·37 | ·9147 | ·77 | ·9616 | ·17 | ·98500 |
| ·18 | ·5714 | ·58 | ·7190 | ·98 | ·8365 | ·38 | ·9162 | ·78 | ·9625 | ·18 | ·98537 |
| ·19 | ·5753 | ·59 | ·7224 | ·99 | ·8389 | ·39 | ·9177 | ·79 | ·9633 | ·19 | ·98574 |
| 0·20 | 0·5793 | 0·60 | 0·7257 | 1·00 | 0·8413 | 1·40 | 0·9192 | 1·80 | 0·9641 | 2·20 | 0·98610 |
| ·21 | ·5832 | ·61 | ·7291 | ·01 | ·8438 | ·41 | ·9207 | ·81 | ·9649 | ·21 | ·98645 |
| ·22 | ·5871 | ·62 | ·7324 | ·02 | ·8461 | ·42 | ·9222 | ·82 | ·9656 | ·22 | ·98679 |
| ·23 | ·5910 | ·63 | ·7357 | ·03 | ·8485 | ·43 | ·9236 | ·83 | ·9664 | ·23 | ·98713 |
| ·24 | ·5948 | ·64 | ·7389 | ·04 | ·8508 | ·44 | ·9251 | ·84 | ·9671 | ·24 | ·98745 |
| 0·25 | 0·5987 | 0·65 | 0·7422 | 1·05 | 0·8531 | 1·45 | 0·9265 | 1·85 | 0·9678 | 2·25 | 0·98778 |
| ·26 | ·6026 | ·66 | ·7454 | ·06 | ·8554 | ·46 | ·9279 | ·86 | ·9686 | ·26 | ·98809 |
| ·27 | ·6064 | ·67 | ·7486 | ·07 | ·8577 | ·47 | ·9292 | ·87 | ·9693 | ·27 | ·98840 |
| ·28 | ·6103 | ·68 | ·7517 | ·08 | ·8599 | ·48 | ·9306 | ·88 | ·9699 | ·28 | ·98870 |
| ·29 | ·6141 | ·69 | ·7549 | ·09 | ·8621 | ·49 | ·9319 | ·89 | ·9706 | ·29 | ·98899 |
| 0·30 | 0·6179 | 0·70 | 0·7580 | 1·10 | 0·8643 | 1·50 | 0·9332 | 1·90 | 0·9713 | 2·30 | 0·98928 |
| ·31 | ·6217 | ·71 | ·7611 | ·11 | ·8665 | ·51 | ·9345 | ·91 | ·9719 | ·31 | ·98956 |
| ·32 | ·6255 | ·72 | ·7642 | ·12 | ·8686 | ·52 | ·9357 | ·92 | ·9726 | ·32 | ·98983 |
| ·33 | ·6293 | ·73 | ·7673 | ·13 | ·8708 | ·53 | ·9370 | ·93 | ·9732 | ·33 | ·99010 |
| ·34 | ·6331 | ·74 | ·7704 | ·14 | ·8729 | ·54 | ·9382 | ·94 | ·9738 | ·34 | ·99036 |
| 0·35 | 0·6368 | 0·75 | 0·7734 | 1·15 | 0·8749 | 1·55 | 0·9394 | 1·95 | 0·9744 | 2·35 | 0·99061 |
| ·36 | ·6406 | ·76 | ·7764 | ·16 | ·8770 | ·56 | ·9406 | ·96 | ·9750 | ·36 | ·99086 |
| ·37 | ·6443 | ·77 | ·7794 | ·17 | ·8790 | ·57 | ·9418 | ·97 | ·9756 | ·37 | ·99111 |
| ·38 | ·6480 | ·78 | ·7823 | ·18 | ·8810 | ·58 | ·9429 | ·98 | ·9761 | ·38 | ·99134 |
| ·39 | ·6517 | ·79 | ·7852 | ·19 | ·8830 | ·59 | ·9441 | ·99 | ·9767 | ·39 | ·99158 |
| 0·40 | 0·6554 | 0·80 | 0·7881 | 1·20 | 0·8849 | 1·60 | 0·9452 | 2·00 | 0·9772 | 2·40 | 0·99180 |

# TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2·40 | 0·99180 | 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 |
| ·41 | ·99202 | ·56 | ·99477 | ·71 | ·99664 | ·86 | ·99788 | ·01 | ·99869 | ·16 | ·99921 |
| ·42 | ·99224 | ·57 | ·99492 | ·72 | ·99674 | ·87 | ·99795 | ·02 | ·99874 | ·17 | ·99924 |
| ·43 | ·99245 | ·58 | ·99506 | ·73 | ·99683 | ·88 | ·99801 | ·03 | ·99878 | ·18 | ·99926 |
| ·44 | ·99266 | ·59 | ·99520 | ·74 | ·99693 | ·89 | ·99807 | ·04 | ·99882 | ·19 | ·99929 |
| 2·45 | 0·99286 | 2·60 | 0·99534 | 2·75 | 0·99702 | 2·90 | 0·99813 | 3·05 | 0·99886 | 3·20 | 0·99931 |
| ·46 | ·99305 | ·61 | ·99547 | ·76 | ·99711 | ·91 | ·99819 | ·06 | ·99889 | ·21 | ·99934 |
| ·47 | ·99324 | ·62 | ·99560 | ·77 | ·99720 | ·92 | ·99825 | ·07 | ·99893 | ·22 | ·99936 |
| ·48 | ·99343 | ·63 | ·99573 | ·78 | ·99728 | ·93 | ·99831 | ·08 | ·99896 | ·23 | ·99938 |
| ·49 | ·99361 | ·64 | ·99585 | ·79 | ·99736 | ·94 | ·99836 | ·09 | ·99900 | ·24 | ·99940 |
| 2·50 | 0·99379 | 2·65 | 0·99598 | 2·80 | 0·99744 | 2·95 | 0·99841 | 3·10 | 0·99903 | 3·25 | 0·99942 |
| ·51 | ·99396 | ·66 | ·99609 | ·81 | ·99752 | ·96 | ·99846 | ·11 | ·99906 | ·26 | ·99944 |
| ·52 | ·99413 | ·67 | ·99621 | ·82 | ·99760 | ·97 | ·99851 | ·12 | ·99910 | ·27 | ·99946 |
| ·53 | ·99430 | ·68 | ·99632 | ·83 | ·99767 | ·98 | ·99856 | ·13 | ·99913 | ·28 | ·99948 |
| ·54 | ·99446 | ·69 | ·99643 | ·84 | ·99774 | ·99 | ·99861 | ·14 | ·99916 | ·29 | ·99950 |
| 2·55 | 0·99461 | 2·70 | 0·99653 | 2·85 | 0·99781 | 3·00 | 0·99865 | 3·15 | 0·99918 | 3·30 | 0·99952 |

The critical table below gives on the left the range of values of $x$ for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

| $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ | $x$ | $\Phi(x)$ |
|---|---|---|---|---|---|---|---|
| | | | 0·9994 | | 0·99990 | | 0·99995 |
| 3·075 | | 3·263 | | 3·731 | | 3·916 | |
| | 0·9990 | | 0·9995 | | 0·99991 | | 0·99996 |
| 3·105 | | 3·320 | | 3·759 | | 3·976 | |
| | 0·9991 | | 0·9996 | | 0·99992 | | 0·99997 |
| 3·138 | | 3·389 | | 3·791 | | 4·055 | |
| | 0·9992 | | 0·9997 | | 0·99993 | | 0·99998 |
| 3·174 | | 3·480 | | 3·826 | | 4·173 | |
| | 0·9993 | | 0·9998 | | 0·99994 | | 0·99999 |
| 3·215 | | 3·615 | | 3·867 | | 4·417 | |
| | 0·9994 | | 0·9999 | | 0·99995 | | 1·00000 |

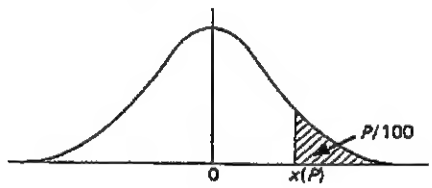
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \dfrac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\dfrac{1}{x^2}+\dfrac{3}{x^4}-\dfrac{15}{x^6}+\dfrac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

# TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}} \int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\, dt.$$

If $X$ is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower $P$ per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



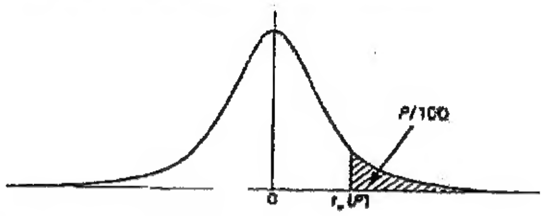

| $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ | $P$ | $x(P)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 50 | 0·0000 | 5·0 | 1·6449 | 3·0 | 1·8808 | 2·0 | 2·0537 | 1·0 | 2·3263 | 0·10 | 3·0902 |
| 45 | 0·1257 | 4·8 | 1·6646 | 2·9 | 1·8957 | 1·9 | 2·0749 | 0·9 | 2·3656 | 0·09 | 3·1214 |
| 40 | 0·2533 | 4·6 | 1·6849 | 2·8 | 1·9110 | 1·8 | 2·0969 | 0·8 | 2·4089 | 0·08 | 3·1559 |
| 35 | 0·3853 | 4·4 | 1·7060 | 2·7 | 1·9268 | 1·7 | 2·1201 | 0·7 | 2·4573 | 0·07 | 3·1947 |
| 30 | 0·5244 | 4·2 | 1·7279 | 2·6 | 1·9431 | 1·6 | 2·1444 | 0·6 | 2·5121 | 0·06 | 3·2389 |
| 25 | 0·6745 | 4·0 | 1·7507 | 2·5 | 1·9600 | 1·5 | 2·1701 | 0·5 | 2·5758 | 0·05 | 3·2905 |
| 20 | 0·8416 | 3·8 | 1·7744 | 2·4 | 1·9774 | 1·4 | 2·1973 | 0·4 | 2·6521 | 0·01 | 3·7190 |
| 15 | 1·0364 | 3·6 | 1·7991 | 2·3 | 1·9954 | 1·3 | 2·2262 | 0·3 | 2·7478 | 0·005 | 3·8906 |
| 10 | 1·2816 | 3·4 | 1·8250 | 2·2 | 2·0141 | 1·2 | 2·2571 | 0·2 | 2·8782 | 0·001 | 4·2649 |
| 5 | 1·6449 | 3·2 | 1·8522 | 2·1 | 2·0335 | 1·1 | 2·2904 | 0·1 | 3·0902 | 0·0005 | 4·4172 |

# TABLE 10. PERCENTAGE POINTS OF THE $t$-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}}\frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)}\int_{t_\nu(P)}^{\infty}\frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let $X_1$ and $X_2$ be independent random variables having a normal distribution with zero mean and unit variance and a $\chi^2$-distribution with $\nu$ degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's $t$-distribution with $\nu$ degrees of freedom, and the probability that $t \geq t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geq t_\nu(P)$ is $2P/100$.




The limiting distribution of $t$ as $\nu$ tends to infinity is the normal distribution with zero mean and unit variance. When $\nu$ is large interpolation in $\nu$ should be harmonic.

| $P$ | 40 | 30 | 25 | 20 | 15 | 10 | 5 | 2·5 | 1 | 0·5 | 0·1 | 0·05 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ = 1 | 0·3249 | 0·7265 | 1·0000 | 1·3764 | 1·963 | 3·078 | 6·314 | 12·71 | 31·82 | 63·66 | 318·3 | 636·6 |
| 2 | 0·2887 | 0·6172 | 0·8165 | 1·0607 | 1·386 | 1·886 | 2·920 | 4·303 | 6·965 | 9·925 | 22·33 | 31·60 |
| 3 | 0·2767 | 0·5844 | 0·7649 | 0·9785 | 1·250 | 1·638 | 2·353 | 3·182 | 4·541 | 5·841 | 10·21 | 12·92 |
| 4 | 0·2707 | 0·5686 | 0·7407 | 0·9410 | 1·190 | 1·533 | 2·132 | 2·776 | 3·747 | 4·604 | 7·173 | 8·610 |
| 5 | 0·2672 | 0·5594 | 0·7267 | 0·9195 | 1·156 | 1·476 | 2·015 | 2·571 | 3·365 | 4·032 | 5·893 | 6·869 |
| 6 | 0·2648 | 0·5534 | 0·7176 | 0·9057 | 1·134 | 1·440 | 1·943 | 2·447 | 3·143 | 3·707 | 5·208 | 5·959 |
| 7 | 0·2632 | 0·5491 | 0·7111 | 0·8960 | 1·119 | 1·415 | 1·895 | 2·365 | 2·998 | 3·499 | 4·785 | 5·408 |
| 8 | 0·2619 | 0·5459 | 0·7064 | 0·8889 | 1·108 | 1·397 | 1·860 | 2·306 | 2·896 | 3·355 | 4·501 | 5·041 |
| 9 | 0·2610 | 0·5435 | 0·7027 | 0·8834 | 1·100 | 1·383 | 1·833 | 2·262 | 2·821 | 3·250 | 4·297 | 4·781 |
| 10 | 0·2602 | 0·5415 | 0·6998 | 0·8791 | 1·093 | 1·372 | 1·812 | 2·228 | 2·764 | 3·169 | 4·144 | 4·587 |
| 11 | 0·2596 | 0·5399 | 0·6974 | 0·8755 | 1·088 | 1·363 | 1·796 | 2·201 | 2·718 | 3·106 | 4·025 | 4·437 |
| 12 | 0·2590 | 0·5386 | 0·6955 | 0·8726 | 1·083 | 1·356 | 1·782 | 2·179 | 2·681 | 3·055 | 3·930 | 4·318 |
| 13 | 0·2586 | 0·5375 | 0·6938 | 0·8702 | 1·079 | 1·350 | 1·771 | 2·160 | 2·650 | 3·012 | 3·852 | 4·221 |
| 14 | 0·2582 | 0·5366 | 0·6924 | 0·8681 | 1·076 | 1·345 | 1·761 | 2·145 | 2·624 | 2·977 | 3·787 | 4·140 |
| 15 | 0·2579 | 0·5357 | 0·6912 | 0·8662 | 1·074 | 1·341 | 1·753 | 2·131 | 2·602 | 2·947 | 3·733 | 4·073 |
| 16 | 0·2576 | 0·5350 | 0·6901 | 0·8647 | 1·071 | 1·337 | 1·746 | 2·120 | 2·583 | 2·921 | 3·686 | 4·015 |
| 17 | 0·2573 | 0·5344 | 0·6892 | 0·8633 | 1·069 | 1·333 | 1·740 | 2·110 | 2·567 | 2·898 | 3·646 | 3·965 |
| 18 | 0·2571 | 0·5338 | 0·6884 | 0·8620 | 1·067 | 1·330 | 1·734 | 2·101 | 2·552 | 2·878 | 3·610 | 3·922 |
| 19 | 0·2569 | 0·5333 | 0·6876 | 0·8610 | 1·066 | 1·328 | 1·729 | 2·093 | 2·539 | 2·861 | 3·579 | 3·883 |
| 20 | 0·2567 | 0·5329 | 0·6870 | 0·8600 | 1·064 | 1·325 | 1·725 | 2·086 | 2·528 | 2·845 | 3·552 | 3·850 |
| 21 | 0·2566 | 0·5325 | 0·6864 | 0·8591 | 1·063 | 1·323 | 1·721 | 2·080 | 2·518 | 2·831 | 3·527 | 3·819 |
| 22 | 0·2564 | 0·5321 | 0·6858 | 0·8583 | 1·061 | 1·321 | 1·717 | 2·074 | 2·508 | 2·819 | 3·505 | 3·792 |
| 23 | 0·2563 | 0·5317 | 0·6853 | 0·8575 | 1·060 | 1·319 | 1·714 | 2·069 | 2·500 | 2·807 | 3·485 | 3·768 |
| 24 | 0·2562 | 0·5314 | 0·6848 | 0·8569 | 1·059 | 1·318 | 1·711 | 2·064 | 2·492 | 2·797 | 3·467 | 3·745 |
| 25 | 0·2561 | 0·5312 | 0·6844 | 0·8562 | 1·058 | 1·316 | 1·708 | 2·060 | 2·485 | 2·787 | 3·450 | 3·725 |
| 26 | 0·2560 | 0·5309 | 0·6840 | 0·8557 | 1·058 | 1·315 | 1·706 | 2·056 | 2·479 | 2·779 | 3·435 | 3·707 |
| 27 | 0·2559 | 0·5306 | 0·6837 | 0·8551 | 1·057 | 1·314 | 1·703 | 2·052 | 2·473 | 2·771 | 3·421 | 3·690 |
| 28 | 0·2558 | 0·5304 | 0·6834 | 0·8546 | 1·056 | 1·313 | 1·701 | 2·048 | 2·467 | 2·763 | 3·408 | 3·674 |
| 29 | 0·2557 | 0·5302 | 0·6830 | 0·8542 | 1·055 | 1·311 | 1·699 | 2·045 | 2·462 | 2·756 | 3·396 | 3·659 |
| 30 | 0·2556 | 0·5300 | 0·6828 | 0·8538 | 1·055 | 1·310 | 1·697 | 2·042 | 2·457 | 2·750 | 3·385 | 3·646 |
| 32 | 0·2555 | 0·5297 | 0·6822 | 0·8530 | 1·054 | 1·309 | 1·694 | 2·037 | 2·449 | 2·738 | 3·365 | 3·622 |
| 34 | 0·2553 | 0·5294 | 0·6818 | 0·8523 | 1·052 | 1·307 | 1·691 | 2·032 | 2·441 | 2·728 | 3·348 | 3·601 |
| 36 | 0·2552 | 0·5291 | 0·6814 | 0·8517 | 1·052 | 1·306 | 1·688 | 2·028 | 2·434 | 2·719 | 3·333 | 3·582 |
| 38 | 0·2551 | 0·5288 | 0·6810 | 0·8512 | 1·051 | 1·304 | 1·686 | 2·024 | 2·429 | 2·712 | 3·319 | 3·566 |
| 40 | 0·2550 | 0·5286 | 0·6807 | 0·8507 | 1·050 | 1·303 | 1·684 | 2·021 | 2·423 | 2·704 | 3·307 | 3·551 |
| 50 | 0·2547 | 0·5278 | 0·6794 | 0·8489 | 1·047 | 1·299 | 1·676 | 2·009 | 2·403 | 2·678 | 3·261 | 3·496 |
| 60 | 0·2545 | 0·5272 | 0·6786 | 0·8477 | 1·045 | 1·296 | 1·671 | 2·000 | 2·390 | 2·660 | 3·232 | 3·460 |
| 120 | 0·2539 | 0·5258 | 0·6765 | 0·8446 | 1·041 | 1·289 | 1·658 | 1·980 | 2·358 | 2·617 | 3·160 | 3·373 |
| ∞ | 0·2533 | 0·5244 | 0·6745 | 0·8416 | 1·036 | 1·282 | 1·645 | 1·960 | 2·326 | 2·576 | 3·090 | 3·291 |